A Level Further Maths

Please complete the below work in preparation for your start at Franklin this September.
This work should be brought with you to enrolment and your first day at Franklin.
If you aren't attending enrolment on site please just bring this when you first come to the college in September.
This should be completed in time for enrolment, work can be in a written or electronic format.
Any questions please contact our admissions department – admissions@franklin.ac.uk

Subject: A Level Maths & Further Maths

Surds, Indices and Algebraic Expressions

You've made a great decision to take this course as part of your study programme at Franklin. Being ready for a flying start in September will really put you ahead of the game, here's how to make a start…

Complete the set of tasks detailed in the table below. If you have any issues, please contact maths@franklin.ac.uk who will be more than willing to assist if you.

Context	The focus of this bridging work is algebra; being able to manipulate and work with algebra fluently will give you a distinct advantage at A level. During your GCSE studies of mathematics, you would have studied algebra in various ways. At A level, algebra underpins the majority of the work you will do, so in order to be successful, you need to have experienced the skills required for the course. You may find some of these topics easy and some challenging, however, it is vital you attempt all of the questions and use the videos to guide you through the more difficult sections. Any 'gaps' you find in your knowledge we would expect you to independently practise those skills before the course. Video links are given throughout each lesson to help you; however, feel free to do your own independent study. There are many excellent YouTube channels and websites out there.
Task	**Surds, Indices and Simple Algebraic Expressions** Complete both Task 1 – Surds and Indices and Task 2 – Simple algebraic Expressions • Use lined paper to complete each exercise. • Neatly and clearly, show all working out (use the examples as a guide). • Use the videos given in the green boxes examples to help you. • Complete the exercise • Mark all your work clearly in a different coloured pen. (Answers at bottom) • Upload your work to Google Classroom.

Task 1 - Surds and Indices

This may seem a rather difficult and even pointless topic when you meet it at GCSE, but you will soon see that it is extremely useful at A Level, and you need to be confident with it.

Watch the following YouTube videos to help you before starting the exercise.

Negative Powers – Exam Solutions - https://www.youtube.com/watch?v=SW9nb-13V6E

Fractional Powers – Exam Solutions - https://www.youtube.com/watch?v=_fadg_VjBMc

Rationalising Surds – Exam Solutions - https://www.youtube.com/watch?v=xehwCkT5aX0

Here are further written examples if you need them after watching the videos. You may also want to use and copy down into your notes.

Examples 1 (a) $\frac{1}{x^3} = x^{-3}$ (b) $\sqrt[3]{x} = x^{\frac{1}{3}}$ (c) $\pi^0 = 1$

(d) $\sqrt[4]{x^7} = x^{7/4}$. The easiest way of seeing this is to write it as $(x^7)^{\frac{1}{4}}$

You will make most use of the rules of ***surds*** when checking your answers! An answer that you give as $\frac{6}{\sqrt{3}}$ will probably be given in the book as $2\sqrt{3}$, and $\frac{2}{3-\sqrt{7}}$ as $3+\sqrt{7}$. Before worrying why you have got these wrong, you should check whether they are equivalent!

Examples 2

Indeed, they are, as

$$\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

and

$$\frac{2}{3-\sqrt{7}} = \frac{2}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{2(3+\sqrt{7})}{3^2-(\sqrt{7})^2} = \frac{2(3+\sqrt{7})}{9-7} = 3+\sqrt{7}.$$

The first of these processes is usually signalled by the instruction “write in surd form” and the second by “rationalise the denominator”.

Remember also that to put a square root in surd form you take out the *biggest* square factor you can. Thus $\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ (noting that you should take out $\sqrt{16}$ and not $\sqrt{4}$).

Exercise: Surds and Indices

1 Write the following as powers of x.

(a) $\frac{1}{x}$ (b) $\frac{1}{x^5}$ (c) $\sqrt[5]{x}$ (d) $\sqrt[3]{x^5}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt{x^3}}$

2 Write the following without negative or fractional powers.

(a) x^{-4} (b) x^0 (c) $x^{1/6}$ (d) $x^{3/4}$ (e) $x^{-3/2}$

3 Write the following in the form ax^n.

(a) $4\sqrt[3]{x}$ (b) $\frac{3}{x^2}$ (c) $\frac{5}{\sqrt{x}}$ (d) $\frac{1}{2x^3}$ (e) 6

4 Write as sums of powers of x.

(a) $x^3\left(x+\frac{1}{x}\right)$ (b) $\frac{x^4+1}{x^2}$ (c) $x^{-5}\left(x+\frac{1}{x^2}\right)$

5 Write the following in surd form.

(a) $\sqrt{75}$ (b) $\sqrt{180}$ (c) $\frac{12}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{3}{\sqrt{12}}$

6 Rationalise the denominators in the following expressions.

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{6}-2}$ (c) $\frac{6}{\sqrt{7}+2}$

(d) $\frac{1}{3+\sqrt{5}}$ (e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

Further Maths Only

7* Simplify $\frac{1}{\sqrt{2}+\sqrt{1}}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{100}+\sqrt{99}}$

Task 2 - Simple Algebraic Expressions

This time we would recommend you take notes on these first few examples before going ot watch the videos highlighted at the bottom of the page. There are some very basic but important things here, but they should prove helpful.

Are you fully aware that $\frac{x}{4}$ and $\frac{1}{4}x$ are the same thing?

Example 1 Find the value of *a* for which $\frac{8}{11}(5x-4)=\frac{8(5x-4)}{a}$ is always true.

Solution Dividing 8 by 11 and multiplying by $(5x-4)$ is the same as multiplying 8 by $(5x-4)$ and dividing by 11. So $a = 11$.

You do not need to multiply anything out to see this!

Remember that in algebraic fractions such as $\frac{3}{x-2}$, the line has the same effect as a bracket round the denominator.

You may well find it helpful actually to *write in* the bracket: $\frac{3}{(x-2)}$.

Example 2 Solve the equation $\frac{3}{x-2}=12.$

Solution

Multiply both sides by $(x-2)$:	$3 = 12(x-2)$
Multiply out the bracket:	$3 = 12x-24$
Add 24 to both sides:	$27 = 12x$
Divide by 12:	$x=\frac{27}{12}=2\frac{1}{4}.$

A common mistake is to start by dividing by 3. That would give $\frac{1}{x-2}=4$ [*not* $x-2=4$] and you will still have to multiply by $(x-2)$.

Don't ever be afraid to get the x-term on the right, as in the last line but one of the working. After all, 27 = 12x means just the same as 12x = 27

Example 3 Solve the equation $\frac{3}{5}(2x+3)=\frac{7}{15}(4x-9)$

Solution Do ***not*** multiply out the brackets to get fractions – that leads to horrible numbers! Instead:

Multiply both sides by 15: $15\times\frac{3}{5}(2x+3)=15\times\frac{7}{15}(4x-9)$

Choose 15 as it gets rid of all the fractions.

Cancel down the fractions: $3\times\frac{3}{1}(2x+3)=\frac{7}{1}(4x-9)$

$$9(2x+3)=7(4x-9)$$

Now multiply out: $18x + 27 = 28x - 63$

$$90 = 10x$$

Hence the answer is $x = 9$

This makes the working very much easier. ***Please don't*** respond by saying "well, my method gets the same answer"! You want to develop your flexibility and your ability to find the easiest method if you are to do well at A Level, as well as to be able to use similar techniques in algebra instead of numbers. It's not just this example we are worried about – it's more complicated examples of a similar type.

YouTube Videos to help you further with the following exercise

Solving Linear Equations - https://www.youtube.com/watch? v=-eNM4GV-X9s

Solving Equations with algebraic fractions - https://www.youtube.com/watch?v=UkboZp0nSQs

Different Types of Equations - https://www.youtube.com/watch?v=_U2arLwb7lk

Rearranging Formulae - https://www.youtube.com/watch?v=9JRXUB2o24Y

https://www.youtube.com/watch?v=qb7qSdmJwT8

Exercise: Simple Algebraic Expressions

1 Find the values of the letters p, q and r that make the following pairs of expressions always equal.

(a) $\frac{1}{7}x = \frac{x}{p}$ (b) $\frac{1}{5}(2x+3) = \frac{(2x+3)}{q}$ (c) $\frac{3}{10}(2-7x) = \frac{3(2-7x)}{r}$

2 Solve the following equations.

(a) $\frac{60}{x+4} = 12$ (b) $\frac{35}{2x-3} = 5$ (c) $\frac{20}{6-x} = \frac{1}{2}$

3 Make $\cos C$ the subject of the formula $c^2 = a^2 + b^2 - 2ab\cos C$.

4 (a) Multiply $\frac{x+5}{4}$ by 8. (b) Multiply $(x + 2) \div 3$ by 12.

(c) Multiply $\frac{1}{2}(x+7)$ by 6. (d) Multiply $\frac{1}{4}(x-3)$ by 8.

5 Solve the following equations (Use example 3 to help).

(a) $\frac{3}{4}(2x+3) = \frac{5}{8}(x-2)$ (b) $\frac{1}{6}(5x+11) = \frac{2}{3}(2x-4)$

(c) $\frac{5}{9}(3x+1) = \frac{7}{12}(2x+1)$

6 Make x the subject of the following equations (These are tricky but give them a go).

(a) $\frac{a}{b}(cx+d) = x+2$ (b) $\frac{a}{b}(cx+d) = \frac{2a}{b^2}(x+2d)$

7 Simplify the following as far as possible.

(a) $\frac{a+a+a+a+a}{5}$ (b) $\frac{b+b+b+b}{b}$

(c) $\frac{c\times c\times c\times c\times c}{c}$ (d) $\frac{d\times d\times d\times d}{4}$

Answers: Simple Algebraic Expressions

1 (a) x^{-1} (b) x^{-5} (c) $x^{1/5}$ (d) $x^{3/5}$ (e) $x^{-1/2}$ (f) $x^{-3/2}$

2 (a) $\frac{1}{x^4}$ (b) 1 (c) $\sqrt[6]{x}$ (d) $\sqrt[4]{x^3}$ (e) $\frac{1}{\sqrt{x^3}}$

3 (a) $4x^{1/3}$ (b) $3x^{-2}$ (c) $5x^{-1/2}$ (d) $\frac{1}{2}x^{-3}$ (e) $6x^0$

4 (a) $x^4 + x^2$ (b) $x^2 + x^{-2}$ (c) $x^{-4} + x^{-7}$

5 (a) $5\sqrt{3}$ (b) $6\sqrt{5}$ (c) $2\sqrt{6}$ (d) $\frac{1}{5}\sqrt{5}$ (e) $\frac{1}{2}\sqrt{3}$

6 (a) $\sqrt{2}+1$ (b) $\sqrt{6}+2$ (c) $2(\sqrt{7}-2)$ (d) $\frac{1}{4}(3-\sqrt{5})$ (e) $\sqrt{6}+\sqrt{5}$

Further Maths only

7* **Looking at Question 6 e):** $\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}+\sqrt{5}}{1} = \sqrt{6}+\sqrt{5}$

Then apply this to question 7*:

$$\frac{1}{\sqrt{2}+\sqrt{1}} = \frac{\sqrt{2}-\sqrt{1}}{1} = \frac{\sqrt{2}-1}{1} = \sqrt{2}-1$$

$$\frac{1}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{1} = \sqrt{3}-\sqrt{2}$$

Etc.

$$-1+\sqrt{100} = 9$$

Answers: Simple Algebraic Expressions

1 (a) $p = 7$ (b) $q = 5$ (c) $r = 10$

2 (a) $x = 1$ (b) $x = 5$ (c) $x = -34$

3 $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

4 (a) $2(x + 5)$ (b) $4(x + 2)$ (c) $3(x + 7)$ (d) $2(x - 3)$

5 (a) $x = -4$ (b) $x = 9$ (c) $x = \frac{1}{18}$

6 (a) $x = \dfrac{2b - ad}{ac - b}$ (or $\dfrac{ad - 2b}{b - ac}$) (b) $x = \dfrac{d(4 - b)}{bc - 2}$ (or $x = \dfrac{d(b - 4)}{2 - bc}$)

7 (a) a (b) 4 (c) c^4 (d) $\frac{1}{4}d^4$